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| ⋮ |
| 00101010 |
| 01101101 |
| 10001001 |
| 10001111 |
| ⋮ |
1. Determine what 4 one-byte integers are stored in these 4 bytes:
 2. Determine what 2 two-byte integers are stored in these 4 bytes.
 3. Determine what four-byte integer is stored in these 4 bytes.
 4. Interpret the 4 bytes as 4 ASCII characters.
 5. Write 6666 in binary form.
 6. Write 6666 in Hex form.
 7. Convert the hex number fafa to decimal form.
 8. Write the polynomial $p(x) = 8x^4 + 4x^3 + 2x^2 - 1$ in Horner's form and use that form to evaluate $p(1.2)$, step by step.
 9. How many multiplications does Horner's method use for this polynomial vs. the method of directly evaluating the polynomial term by term?
 10. Write a C++ program to solve the equation $ax + b = c$ where the user supplies a , b and c and the program returns the solution, if there is one, or an error message otherwise. Submit the program using <your initials>lab2.cpp to ghagopian@collegeofthedesert.edu

SOLUTIONS:

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| ⋮ |
| 00101010 |
| 01101101 |
| 10001001 |
| 10001111 |
| ⋮ |
1. Determine what 4 one-byte integers are stored in these 4 bytes:
 $00101010_2 = 2^5 + 2^3 + 2^1 = 32 + 8 + 2 = 42_{10}$
 $01101101_2 = 2^6 + 2^5 + 2^3 + 2^2 + 2^0 = 64 + 32 + 8 + 4 + 1 = 109_{10}$
 $10001001_2 = 2^7 + 2^3 + 2^0 = 128 + 8 + 1 = 137_{10}$
 $10001111_2 = 2^7 + 2^3 + 2^2 + 2^1 + 2^0 = 128 + 8 + 4 + 2 + 1 = 143_{10}$
 2. Determine what 2 two-byte integers are stored in these 4 bytes.
 ANS: This depends on which is the more significant byte.
 Thus the first two bytes could be interpreted as either
 $0010101001101101_2 = 42 * 2^8 + 109 = 10861$ or
 $0110110100101010_2 = 109 * 2^8 + 42 = 27946$
 and the second two bytes could be
 $1000100110001111_2 = 137 * 2^8 + 143 = 35215$ or
 $1000111110001001_2 = 143 * 2^8 + 137 = 36745$
 3. Determine what four-byte integer is stored in these 4 bytes.
 ANS: Again there are two reasonable interpretations, depending on whether the first or last byte is the most significant:
 $00101010011011011000100110001111_2 = 10861 * 2^{16} + 35215 = 711821711$ or
 $10001111100010010110110100101010_2 = 36745 * 2^{16} + 27946 = 2408148266$

4. Interpret the 4 bytes as 4 ASCII characters.

ANS: The original ASCII character set had only 128 characters, and the printable characters were from 32–126. Going to the standard extended ASCII table or at the site http://en.wikipedia.org/wiki/Code_page_437 we can look up the values. Note that, along with the characters in the range 0 to 31 (00_{hex} to 1F_{hex}), which can be interpreted as ASCII controls as well as graphical “dingbats,” some characters have overloaded meanings.

Executing the following code snippet gives a good idea of how our C++ compiler interprets these:

```
cout << char(42) << endl; //asterisk
cout << char(109) << endl; // m
cout << char(137) << endl; // (e-umlaut or diaeresis)
cout << char(143) << endl; // A with a little circle above
we see that
char(42)=*, char(109)=m, char(137)=ë and char(143)=Å
```

5. Write 6666 in binary form.

ANS: The simplest algorithm for converting from decimal to binary is Horner’s algorithm. To illustrate how this works, consider the number

$$37 = 32 + 4 + 1 = 1*2^5 + 0*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 1*2^0.$$

Horner’s form for this is $(((((0*2+1)*2 + 0)*2 + 0)*2 + 1)*2 + 0)*2 + 1$.

So we note that the number is odd by putting down a “1”, subtracting one and dividing by 2.

Now we have

$$18 = 16 + 2 = 1*2^4 + 0*2^3 + 0*2^2 + 1*2^1 + 0*2^0 = (((((0*2+1)*2 + 0)*2 + 0)*2 + 0)*2 + 1)*2 + 0.$$

We note that this is even so put down a zero, for “01” and divide by 2:

$$9 = 8 + 1 = 1*2^3 + 0*2^2 + 0*2^1 + 1*2^0 = (((0*2+1)*2 + 0)*2 + 0)*2 + 1$$

This is odd so put down a one for “101” and repeat division by 2:

$$4 = 1*2^2 + 0*2^1 + 0*2^0 = ((0*2+1)*2 + 0)*2 + 0$$

Again it’s even so we have “0101” and dividing by 2:

$$2 = 1*2^1 + 0*2^0 = (0*2+1)*2 + 0$$

Even again so we get “00101” and divide by 2:

$$1 = 1*2^0 = 0*2+1$$

And then finally, “100101” is our binary number.

Applying this algorithm to 6666, the results are tabulated at right, showing that $6666 = 1101001100010_2 = 2^{12} + 2^{11} + 2^9 + 2^6 + 2^5 + 2$ or $4096 + 2048 + 512 + 8 + 2$

| k | x | b_k |
|-----|------|-------|
| 0 | 6666 | 0 |
| 1 | 3333 | 1 |
| 2 | 1666 | 0 |
| 3 | 833 | 1 |
| 4 | 416 | 0 |
| 5 | 208 | 0 |
| 6 | 104 | 0 |
| 7 | 52 | 0 |
| 8 | 26 | 0 |
| 9 | 13 | 1 |
| 10 | 6 | 0 |
| 11 | 3 | 1 |
| 12 | 1 | 1 |
| | 0 | |

6. Write 6666 in Hex form.

ANS: Use algorithm 1.5 from the text: Decimal Integer to Hexadecimal

To convert the integer x into its equivalent hexadecimal numeral:

1. Assert $x > 0$.
2. Set $x = 0$.
3. Divide x by 16, setting x equal to the (integer) quotient.
4. Set h_k equal to the remainder from the previous division.
Use one of the 16 **hexadecimal digits** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f, representing the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, for h_k .
5. Add 1 to k .
6. If $x > 0$, repeat steps 3-6.
7. Return $h_k \cdots h_2 h_1 h_0$ (i.e., the hexadecimal numeral whose j th hex symbol is h_j .)

| k | x | Quotient after division by 16 | Remainder after division by 16 |
|-----|------|-------------------------------|--------------------------------|
| 0 | 6666 | 416 | 10 = a |
| 1 | 416 | 26 | 0 |
| 2 | 26 | 1 | 10 = a |
| 3 | 1 | 0 | 1 |

This is then $1 \times 16^3 + 10 \times 16^2 + 0 \times 16^1 + 10 \times 16^0 = 4096 + 2560 + 10 = 6666$

7. Convert the hex number fafa to decimal form.

ANS: Since f is hex for 15 and a is hex for 10, this is $1a0a_{16} =$

$$15 \times 16^3 + 10 \times 16^2 + 15 \times 16^1 + 10 \times 16^0 = 61440 + 2560 + 240 + 10 = 64250$$

8. Write the polynomial $p(x) = 8x^4 + 4x^3 + 2x^2 - 1$ in Horner's form and use that form to evaluate $p(1.2)$, step by step.

ANS: $p(x) = 8x^4 + 4x^3 + 2x^2 - 1 = (((8x + 4)x + 2)x + 0)x - 1$

$$\begin{aligned} \text{So } p(1.2) &= (((8 \cdot 1.2 + 4) \cdot 1.2 + 2) \cdot 1.2 + 0) \cdot 1.2 - 1 = (((13.6) \cdot 1.2 + 2) \cdot 1.2 + 0) \cdot 1.2 - 1 \\ &= ((18.32) \cdot 1.2 + 0) \cdot 1.2 - 1 \\ &= (21.984) \cdot 1.2 - 1 \\ &= 25.3808 \end{aligned}$$

9. How many multiplications does Horner's method use for this polynomial vs. the method of directly evaluating the polynomial term by term?

ANS: The $p(x) = 8 \cdot x \cdot x \cdot x \cdot x + 4 \cdot x \cdot x \cdot x + 2 \cdot x \cdot x - 1$ form requires 9 multiplications whereas $p(1.2) = (((8 \cdot 1.2 + 4) \cdot 1.2 + 2) \cdot 1.2 + 0) \cdot 1.2 - 1$ requires only 4.